A STUDY OF THE TURBULENT CHARACTERISTICS OF A FREE CIRCULAR JET OF AN INCOMPRESSIBLE GAS

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Inzhenerno-Fizicheskii Zhurnal, Vol. 14, No. 3, pp. 415–422, 1968

UDC 532.517.4

Results are presented for the measurement of the velocity, temperature, and intensity of variations in these quantities within a free circular jet. Data are presented on the distribution law for these variations and for their derivatives with respect to time, as well as on the autocorrelation and spectral functions of the fluxuations in velocity and temperature.

We measured the turbulent characteristics for a jet of air discharging at a velocity of 35 m/sec(Re $\approx$ $\approx 70 \cdot 10^3$) from a nozzle 30 mm in diameter. The stream of air was heated to a maximum of 60° C over the temperature of the ambient medium.

We determined the velocity distribution in the jet from the dynamic head, measured by means of a Pitot tube 0.35 × 0.07 mm in size, and with the aid of a micromanometer. The mean temperatures at various points in the jet were measured by means of a chromel-alumel thermocouple whose junction exhibited an outside diameter of 0.2 mm, and also with a PP potentiometer. The turbulent fluctuations of velocity and temperature within the jet were determined with a thermoanemometer. A single-filament tungsten-wire sensing element was used as the sensor. For purposes of measuring the velocity fluctuations, the sensor was fabricated out of a wire 20 μm in diameter and 4.5 mm in length; to measure the fluctuations in temperature, the sensor was made of a wire 5 μm in diameter (with the sensing portion 1.5 mm in length); the remaining portion of the sensor filament was coated with copper to a diameter of 40 μm.

A positioning device was used to move the sensors within the flow, thus making it possible to determine the magnitudes of this displacement to an accuracy of 0.05 mm.

The mean and fluctuating magnitudes of the velocity and temperature were measured along the stream axis (x/d = 0–10) and at the lateral cross section of the jet, at a distance of 10 diameters from the nozzle outlet.

The velocity characteristics were measured with a ETA-5A electrothermoanemometer which operated at the constant temperature of the sensor filament. The constant-temperature method is particularly expedient in measurements for jets in which the fluctuations are particularly intense, thus making it necessary to account for the nonlinearity of the velocity characteristic of the sensor. Moreover, under conditions of high-intensity jet-velocity fluctuations, the constant-temperature regime provides for a more qualitative compensation of the sensor-filament time constant than the regime of constant filament heating.

The sensor filament was heated to 150° C above the flow temperature; the steepness of the anemometer circuit was 120 A/V. The time constant of the center for these conditions of anemometer-circuit operation was diminished by a factor of 250 relative to its value with the constant-current method and amounted to 10^{-5} sec.

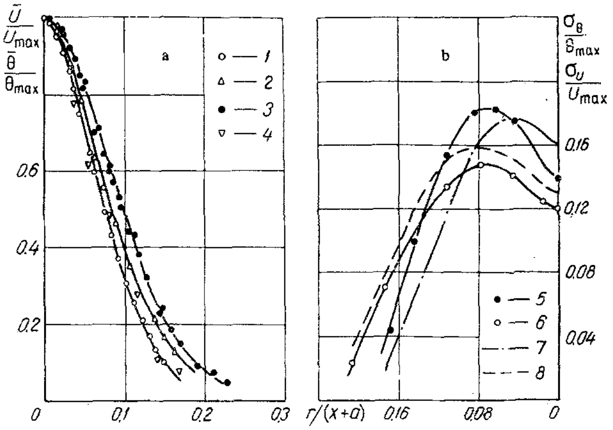

Fig. 1. Distribution of mean velocity, temperature, and intensity of turbulent fluctuations in the jet cross section x/d = 10: 1) $\bar{U}/U_{max}$ in isothermal jet; 2) $\bar{U}/U_{max}$ in heated jet; 3) θ/θ_{max}; 4) $\bar{U}/U_{max}$ from thermoanemometer measurement; 5) σ_U/U_{max}; 6) $\sigma_\theta/\theta_{max}$; 7) σ_U/U_{max} according to [4]; 8) $\sigma_\theta/\theta_{max}$ after [5].

The calibration of the sensors showed that the heat-balance equation for the filament can be rather accurately approximated by an expression of the form

$$\bar{U}^{0.5} = B + CI^2. \tag{1}$$

Equation (1) was subsequently taken into consideration in the determination of the mean value and of the intensity of velocity fluctuation from the oscillogram of the anemometer-bridge current.

For purposes of recording only the temperature variations within the flow, we employed a thermoanemometer circuit which worked on the constant-current method, since the anemometer filament (operating at constant temperature) retains greater sensitivity to velocity fluctuations when the fluctuations in temperature are substantial. For the measurement of the temperature fluctuations, the filament was superheated by $\approx$0.1° C, and the time constant was $4 \cdot 10^{-4}$ sec. The sensitivity of the sensors to a change in temperature was determined by calibration.

An MPO-2 loop oscillograph (loop IV) was used to record the current of the thermoanemometer bridge

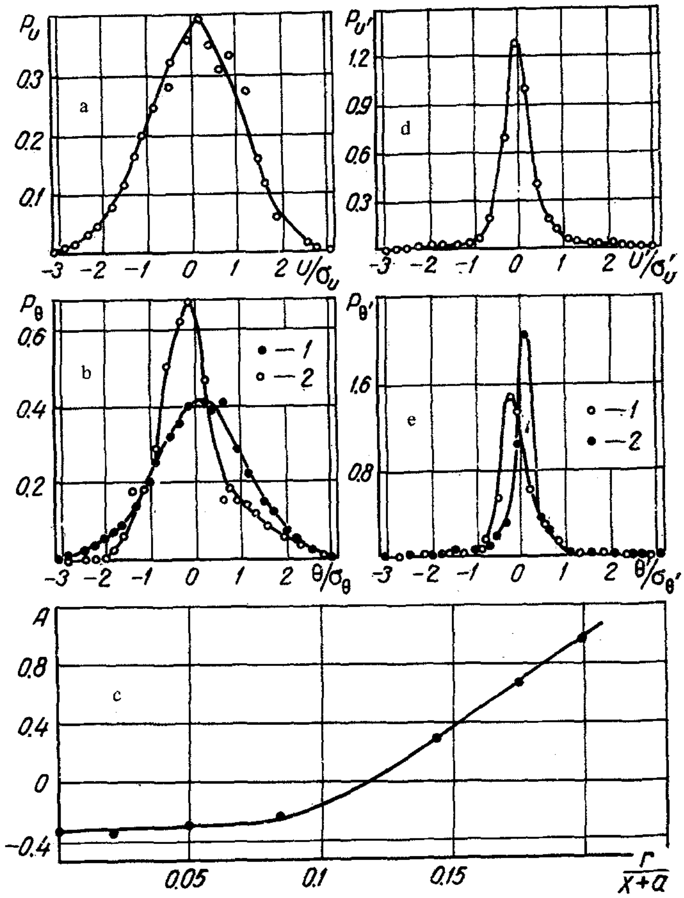

Fig. 2. Curves for the probability density of the fluctuations and their derivatives in the jet cross section $x/d = 10$: a) P_U for $r/(x + a) = 0$; $A = -0.272$; $E = -0.11$; b) P_θ; 1) $r/(x + a) = 0$; $A = -0.354$; $E = 0.52$; 2) $r/(x + a) = 0.2$; $A = 1.13$; $E = 3.63$; c) variation in the asymmetry coefficient in distribution of temperature-fluctuation probabilities; d) $P_{U'}$ for $r/(x + a) = 0$; e) $P_{\theta'}$; 1 $r/(x + a) = 0$; 2) $r/(x + a) = 0.2$.

during the measurement of the velocity and voltages applied to the sensor in the measurement of the temperature fluctuations; this device made provision for motion picture film recording or for the use of a correlometer which is a type of high-speed analog computer. The correlometer described in [1] was used to find the mean-square values of the temperature fluctuations. as well as the fluctuations in velocity over the range in mean velocity values (25–35 m/sec) in which the relationship between the heating current for the filament and the flow velocity is close to the linear.

The experimental data obtained in the form of an oscillogram on the film were converted into punch-card code notation and then processed on an electronic digital computer.

The computation program made it possible to derive the following characteristics of a stationary random function: the mathematical expectation, the standard deviation, the probability distribution for the amplitudes of the fluctuations, the asymmetry and excess parameters, the autocorrelation and spectral functions, and finally, the distribution of the probabilities for the first derivatives of the velocity and temperature fluctuations. The program made provision for the linearization of the anemometer-sensor characteristics on the basis of Eq. (1) (the anemometer calibration curve), and thus ensured computational reliability for the above-indicated characteristics of the velocity fluctuations for the entire range of velocities.

The measured velocity and temperature profiles for the flow at the nozzle outlet were flat. The intensity of turbulence within the flow at the nozzle outlet was insignificant ($\sigma_U/U < 0.01$).

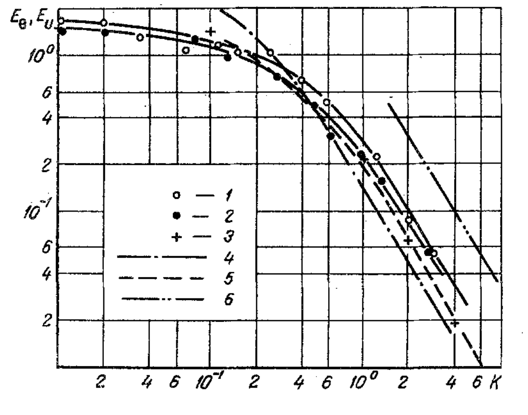

Fig. 3. Energy spectrum of velocity and temperature fluctuations on the jet axis at the cross section x/d = 10: 1) E_U; 2) E_θ; 3) calculation of E_U according to the autocorrelation curve; 4) E_U after [9]; 5) E_θ after [9]; 6) straight line with angular coefficient of –5/3.

The profiles for the average values of the velocity and temperature in the lateral cross section of the jet (x/d = 10) are well-generalized by a relationship of the Gaussian-curve type (Fig. 1a). The data obtained with the thermoanemometer agree with the Pitot tube measurements.

A comparison of the profiles shows that the heated jet spreads out over a wider area than the isothermal jet, i.e., the turbulent mixing is more intense when we have differences in the gas densities. Moreover,

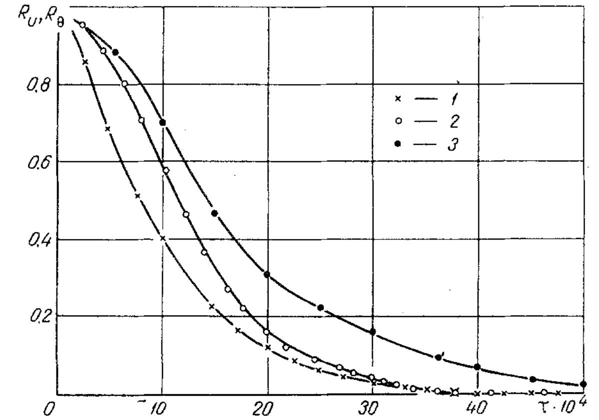

Fig. 4. Autocorrelation functions for turbulent velocity and temperature fluctuations in the cross section x/d = 10 (τ, in sec): 1) R_U for $r/(x + a) = 0$; 2) R_θ for $r/(x + a) = 0$; 3) R_θ for $r/(x + a) = 0.08$.

the coefficient of turbulent thermal diffusivity is somewhat higher than the coefficient of turbulent viscosity, thus yielding a temperature profile that is somewhat fuller than the velocity profile. The convergence of the curves in the peripheral zone of the jet shows that the ratio of the coefficients is apparently variable over the cross section. According to analysis [2] and according to the experiments carried out in [3] with liquids exhibiting substantial divergent physical properties, the ratio of the coefficients of turbulent thermal diffusivity and turbulent viscosity in the core of the flow is close to 1.4.

The change in the intensity of the fluctuation in the longitudinal velocity component across the flow in the case of isothermal flow and the change in the fluctuations of temperature in a heated jet are quite similar (Fig. 1b). The intensity maxima are found in the zone which the flow exhibits the greates values for the gradients of the averaged fields.

These results are in close agreement with the Corrsin data [4, 5]. The existing divergence in the relative intensities of the velocity fluctuations—reaching 30% in the peripheral portion of the jet ($r/(x + a) = 0.12$–0.17)—is evidently associated with the differing measurement methods. According to [2], the constant-current method in the anemometer operation employed by Corrsin can yield a significant understatement of the intensity of velocity fluctuation relative to the constant filament-temperature method, when we consider the nonlinearity of the calibration curve which was employed in these experiments.

In measuring the intensities of the temperature fluctuations we employed a method identical to the one used in [5], i.e., the sensor filaments functioned as resistance thermometers. This portion of the results is in good agreement with the Corrsin measurements.

A significant divergence is found for the results obtained from the data of [6], both in terms of the velocity-fluctuation intensity and the intensity of the temperature variation.

According to the experimental data, at a distance of about 8d from the nozzle outlet the distribution of the mean velocities and the distribution of the temperatures in the jet become similar. For the distribution of the fluctuation intensities σ_U/U_{max} and $\sigma_\theta/\theta_{max}$ we find no tendency toward self-similarity in the measurement segment (x/d = 0–10). The relative intensity of the velocity fluctuations increases over the entire segment; the intensity of the temperature fluctuations at first increases and then, beginning from x/d = 9, we find a tendency toward reduction.

From the probability standpoint, the most complete characteristic of the random function is the distribution law. For isotropic turbulence, we established that the Gaussian curve serves as an excellent approximation of the distribution curve for the components of the velocity fluctuations. Thus according to the Townsend [7] data the distribution is symmetrical and exhibits an excess parameter in the range 2.99–3. The probability density given in [8] for the temperature fluctuations in the wake behind a heat source in an isotropic flow deviates markedly from the normal law.

Of particular interest is the determination of the probability density for turbulence with a lateral shift, where the effect of the velocity and temperature gradient may make itself felt. By processing the derived oscillograms of the velocity and temperature fluctuations, we were able to determine the distribution densities at several points of the lateral cross section of the jet. The distributions of the fluctuation amplitudes at various points in the core of the flow ($0 \le r/(x + a) \le 0.1$) (Fig. 2a and b) do not exhibit significant differences. The statistical distribution is evened out well by the Charlier curve. With increasing distance from the core ($r/(x + a) > 0.1$) the excess coefficient increases and the sign of the asymmetry coefficient changes.

The change in the sign of the asymmetry (Fig. 2c) is apparently associated with the predominant generation of intensive large-scale vortices within a specific region of the jet in which the coefficient A is close to zero ($r/(x + a) \approx 0.1$). The penetration of such vortices to the center of the jet and to the periphery leads to intensive fluctuations of correspondingly different signs, which is consequently reflected in the sign of the asymmetry coefficient.

The probability densities of the first derivatives with respect to time for the fluctuations of the velocity and temperature components differ markedly from the normal law (Fig. 2d and e), whereby we note a substantial increase in the probability density of the zero values of the derivative in the peripheral zone of the jet in comparison with the core. This type of probability distribution of U and θ and their derivatives is associated with the intermittence of the flow.

The energy distribution for the turbulent fluctuations with respect to frequencies, or with respect to the wave numbers, was determined by passing a signal through the narrow-band filter (a relative passband of 3%) and by the measurement of the mean-square magnitude of this signal after the filter by means of the correlometer. From the derived values of the intensities we subsequently determined the variance density for the chosen quasi-resonance frequency. The infrasonic-frequency analyzer developed by the L'vov Polytechnic Institute served as the band filter. The frequency range of the filter was 0.5–1000 Hz.

In determining the spectral density of the temperature fluctuations—when the thermoanemometer was working in accordance with the constant-current method—we introduced a correction factor for the effect of the time constant of the anemometer sensor filament. To verify the measurements and the standardization of the results, we used the relationship

$$\sigma_{U,\theta} = \left(\int_0^\infty E_{U,\theta}\, df \right)^{0.5}. \qquad (2)$$

The difference between the right- and left-hand members of Eq. (2) was insignificant. In particular, in the determination of the one-dimensional spectrum of the velocity fluctuations, the divergence amounted to 4%.

The spectrum of the longitudinal velocity fluctuations and of the temperature fluctuations on the jet axis was taken for a frequency range of 4–1000 Hz (wave numbers of $1.04 \cdot 10^{-2}$–$2.6\ \mathrm{cm}^{-1}$) (Fig. 3). Analysis of the curves shows that 75% of the fluctuation energy is concentrated in the wave-number range up to $1\ \mathrm{cm}^{-1}$. There is no significant difference between the spectra of the velocity and temperature fluctuations. Both of the spectra in the region of large wave numbers are close to the Kolmogorov spectral law (–5/3). In comparison with the results of Corrsin [9], the spectral-density measurements which we carried out are broadened by an order of magnitude in the region of lower wave numbers corresponding to energy-containing vortices. In both of the wave-number ranges, the comparable spectra exhibit no significant divergences.

Processing of the oscillograms gave us one of the integral characteristics which reflected the interrelationship of the time variations—the normed autocorrelation function (Fig. 4).

The autocorrelation functions of the pulsation components of U and θ at all points of the flow can be approximated by an exponential curve. The time correlations for the central portion of the jet ($r/(x + a) \le 0.05$) are close to those represented in the figure by the autocorrelation curves of the fluctuations on the jet axis. We find some divergence between the correlation functions of the velocity and temperature fluctuations for values of $\tau < 1.5 \cdot 10^{-3}$, whereas the spectral densities of these fluctuations are close to one another. This is apparently associated with the effect of the thermal inerta of the sensor in measuring the temperature fluctuations. The effect of the time constant for the sensor filament in the determination of the autocorrelation function of the temperature fluctuations was not taken into consideration; the inertia was taken into consideration in the analysis of the spectrum.

The time characteristics of turbulence can be found from the autocorrelation curve. The magnitude of the largest time relationship for the fluctuations (macroscale) can be assumed equal to $T_e = 1.2 \cdot 10^{-3}$ sec; the time segment $\tau_e = 0.7 \times 10^{-3}$ sec corresponds to the most rapid changes in the fluctuation magnitudes on the axis of the jet (Euler microscale). This small difference between the macro- and micro-scales indicates the comparatively uniform structure of the turbulent vortices and the temperature perturbations.

For points in the jet removed from the axis by more then $r/(x + a) = 0.05$, the form of the correlation curve is flatter and the fraction of the low-frequency fluctuation energy increases.

The spectral function can be determined by Fourier transformation of the autocorrelation function. Figure 3 shows the distribution of the spectral density for the velocity fluctuations on the jet axis, found in this manner. For a wave number K = 0 the spectral density was 1.71 cm. The values of the spectral density, calculated from the autocorrelation function and derived by measurement with the aid of the band filter, are in good agreement.

NOTATION

d is the nozzle diameter; x is the coordinate along the jet axis; r is the radius of the measurement point a is the distance from the nozzle outlet section to the conditional discharge source; $\overline{U}$ is the longitudinal velocity; U is the longitudinal component of the fluctuation velocity; U_{max} is the velocity at the jet axis; $\overline{\theta}$ is the excess temperature at a point in the flow; θ_{max} is the excess temperature at the jet axis; θ is the temperature fluctuation; σ_U is the intensity of the longitudinal velocity fluctuations; σ_θ is the intensity of the temperature fluctuations; P_U and $P_{U'}$ are, respectively, the probability densities for the velocity fluctuations and its time derivative; P_θ and $P_{\theta'}$ are, respectively, the probability density for the temperature fluctuations and its time derivative; E is the excess coefficient; A is the asymmetry coefficient; R_U and R_θ are, respectively, the autocorrelation coefficients for the velocity and temperature fluctuations; T_e is the Euler time macroscale; τ_e is the Euler time microscale; E_U and E_θ are, respectively, the spectral densities for the turbulent fluctuations in velocity and temperature; f is the frequency; $K = 2\pi f/U$ is the wave number; I is the filament-heating current; τ is the time; Re is the Reynolds number.

REFERENCES

1. V. P. Bobkov, Yu. I. Gribanov, M. Kh. Ibragimov, E. V. Nomofilov, and V. I. Subbotin, Teplofizika vysokikh temperatur, 3, no. 5, 1965.

2. I. O. Hintze, Turbulence [Russian translation], Fizmatgiz, 1963.

3. Z. B. Sakipov and D. Zh. Temirbaev, Teplo- i massoperenos, 2, 1965.

4. S. Corrsin, NACA, Rep. W-94, 1943.

5. S. Corrsin and M. S. Uberoi, NACA [in Russian],, Rep. no. 998, 1950.

6. G. S. Antonova, Transactions of the Conference on Applied Gasdynamics [in Russian], Alma-Ata, 1959.

7. A. A. Townsend, Proc. Cambridge Phil. Soc. 43, pt. 40, Oct. 1947.

8. M. S. Uberoi and S. Corrsin, NACA, Rep. 1942, 1952.

9. S. Corrsin and M. S. Uberoi, NACA, Rep. 1040, 1941.

20 June 1967

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